

## AEROELASTIC TIME RESPONSE ANALYSIS OF THIN AIRFOILS BY TRANSONIC CODE LTRAN2

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**Abstract**—A computer based numerical procedure is used to perform aeroelastic time response analysis of thin airfoils oscillating with single and two d.o.f.'s in two-dimensional, unsteady, low-frequency, small-disturbance, transonic potential flow. A computational transonic code LTRAN2 based on fully implicit time integration scheme is employed to obtain the aerodynamic forces. Structural equations of motion and the aerodynamic equations are integrated simultaneously by a numerical method. Results are first obtained for a time response example for a flat plate oscillating at  $M_\infty = 0.7$  with (1) a single pitching d.o.f.; (2) a single plunging d.o.f.; and (3) two d.o.f.s—plunging and pitching. A parallel set of results are also obtained based on subsonic aerodynamic theory for comparison. The agreement is good. Results are then obtained for a time response example for a NACA 64A006 airfoil pitching and plunging at  $M_\infty = 0.85$  which was found to give the lowest flutter speed for the case considered. The parameters that result in neutrally stable response agree with those found in a separate flutter analysis. It was found through comparison between flutter results and response analysis that the principle of superposition of airloads is valid for the present example cases. The results also include the effect of altitude on aeroelastic response.

### NOMENCLATURE

- $a_h$  distance between midchord and elastic axis measured in semi-chords, positive towards the trailing edge
- $A_1$   $C_d/I_a\omega$ , damping parameter for single degree of freedom pitching case
- $A_2$   $K_a/I_a\omega^2$ , stiffness parameter for single degree of freedom pitching case
- $b$  semi-chord length
- $B_1$   $C_h/m\omega$ , damping parameter for single degree of freedom plunging case
- $B_2$   $K_h/m\omega^2$ , stiffness parameter for single degree of freedom plunging case
- $C_h$  mechanical damping coefficient corresponding to plunging
- $C_a$  mechanical damping coefficient corresponding to pitching
- $c$  full-chord length
- $c_l$  total aerodynamic lifting force coefficient
- $c_m$  total aerodynamic moment coefficient about pitching axis
- $c_{l\delta}$  aerodynamic lifting force coefficient due to unit change in plunging displacement  $\delta$
- $c_{l\alpha}$  aerodynamic lifting force coefficient due to unit change in pitching angle  $\alpha$
- $c_{m\delta}$  aerodynamic moment coefficient about pitching axis due to unit change in plunging displacement  $\delta$
- $c_{m\alpha}$  aerodynamic moment coefficient about pitching axis due to unit change in pitching angle  $\alpha$
- $h$  plunging degree of freedom positive when measured downwards
- $I_a$  polar moment of inertia of airfoil mass about the elastic axis
- $k_b$   $\omega b/U$ , reduced frequency with respect to semi-chord length  $b$
- $k_c$   $\omega c/U$ , reduced frequency with respect to full-length chord length  $c$
- $K_h$  bending stiffness coefficient corresponding to plunging displacement
- $K_a$  torsional stiffness coefficient corresponding to pitching rotation
- $m$  mass of the airfoil per unit span
- $M_\infty$  free stream Mach number
- $\{p\}$  aerodynamic load vector
- $q$   $\rho U^2/2$ , dynamic pressure
- $Q_h$   $q c c_{l\delta}$ , total aerodynamic lifting force, positive in  $+h$  direction
- $Q_a$   $q c^2 c_{m\delta}$ , total aerodynamic moment about pitching axis, positive in  $+\alpha$  direction
- $r_a$   $(I_a/m b^2)^{1/2}$ , radius of gyration about elastic axis in semi-chords
- $S$  airfoil static moment about elastic axis
- $t$  time in seconds
- $\bar{t}$   $\omega t$ , nondimensional time parameter
- $\{u\}$  displacement vector
- $U$  free stream velocity
- $U^*$   $U/b\omega$ , nondimensional flight speed parameter
- $x_a$   $S/m b$ , distance between the elastic axis and the mass center measured in semi-chord, positive towards the trailing edge
- $\alpha$  pitching degree of freedom measured positive in nose up direction
- $\alpha_i$  induced angle of attack
- $\beta$   $1/(1 - M_\infty^2)^{1/2}$ , Prandtl-Glauert number
- $\gamma$  ratio of specific heats
- $\delta$   $h/c$ , nondimensional plunging displacement
- $\zeta_h$   $C_h/m\omega$ , uncoupled mechanical damping parameter corresponding to plunging in two degree of freedom system
- $\zeta_a$   $C_a/m b^2\omega$ , uncoupled mechanical damping parameter corresponding to pitching in two degree of freedom system

- $\mu$   $m/\pi pb^2$ , airfoil-air mass ratio
- $\mu'$   $I_a/\pi pb^4$ , airfoil-air mass moment of inertia ratio
- $\xi$   $h/b$ , nondimensional plunging displacement
- $\rho$  free stream air density
- $\omega$  frequency of harmonic oscillation
- $\omega_h$   $(K_h/m)^{1/2}$ , uncoupled plunging frequency of airfoil
- $\omega_a$   $(K_a/I_a)^{1/2}$ , uncoupled pitching frequency of airfoil

## INTRODUCTION

In recent years, there has been extensive developments in unsteady computational aerodynamics. Several large computational programs have been developed. A state-of-the-art review of numerical methods for unsteady aerodynamics was given by Ballhaus and Bridgeman[1].

Based on the developments in computational aerodynamics, aeroelastic applications have followed. In this study, an application of computational transonic aerodynamics to time-dependent aeroelastic analysis is carried out.

Time dependent aeroelastic response analysis of aeroelastic systems is important in the field of aircraft design. Such analysis usually involves in the study of the response motions of an aeroelastic system which varies with time. The type of resulting motion depends on the initial conditions, the characteristics of applied forces and the dynamic response properties of the structural system. Rapidly applied external forces which cause such motion may arise from sources such as gust. The nature of aerodynamic forces during such motion depend on flow conditions which may be either subsonic, transonic or supersonic.

A time-dependent response study requires simultaneous integration of the structural and aerodynamic equations in time. In the subsonic and supersonic cases, the governing flow equations are linear and aerodynamic forces depend upon the body motion in a linear fashion. Many unsteady aerodynamic theories are available for the solution of the aeroelastic problems for these cases. Several classical examples, illustrating the dynamic response phenomena for the subsonic case may be found, for example, in [2].

On the other hand, the governing equations for flows in the transonic regime are nonlinear and they are characterized by the presence of shocks on the airfoil. Because of these complexities, the study of the aeroelastic phenomenon in the transonic regime is fairly involved. From earlier studies it has been observed that for the case of flow over an airfoil in a free stream at Mach numbers near one, small amplitude motions can cause large variations in the aerodynamic forces and moments. In addition, phase differences between the flow variables and the resultant forces may be significant. These special characteristics of the transonic flows increase the possibility of encountering aerolastic instabilities.

In computational aerodynamics, methods for the computation of aerodynamic forces of small-disturbance transonic flows about oscillating airfoils and planar wings have been well developed. A brief survey of such developments can be found in [3]. Based on these developments aeroelastic computations have been carried out. A state-of-the-art review of the developments in transonic aeroelasticity was given by Ashley[4].

The first application of unsteady transonic computations to an aeroelastic response study was by Ballhaus and Goorjian[5]. They illustrated the use of the time-integration and the indicial approaches for the aeroelastic response problems. They performed an aeroelastic response analysis for a NACA 64A006 airfoil with a single pitching degree of freedom with pitching axis at the mid-chord. The time responses were computed by using their program LTRAN2 for unsteady transonic flow coupled with an integration procedure for the differential equation of motion for the airfoil. The motion of the airfoil in a flow at  $M_\infty = 0.88$  was forced for the first few cycles with an amplitude of oscillation of 0.5 degrees until the pitching moment became periodic, after which the airfoil motion and the aerodynamic response were left free to drive each other. As the structural damping coefficient was varied parametrically, the responses were obtained for the highly damped, neutral, slightly divergent and highly divergent cases. It was shown that in order to obtain a flutter solution for their example system at  $M_\infty \geq 0.88$ , it is necessary that the nonlinear aerodynamic equations be used and that the moment variation lead the motion. Linear flat plate equations can not be used to predict a phase lead and thus can not be used to obtain a flutter solution.

Ballhaus and Goorjian also considered another case with a large initial amplitude of  $1.5^\circ$  and  $M_\infty = 0.86$ . This example illustrated a nonsinusoidal pitching moment behavior that could result from a large shock wave excursions encountered at larger airfoil motion amplitudes.

Rizzetta[6] performed an aeroelastic response study of a NACA 64A010 airfoil by simultaneously integrating the LTRAN2 aerodynamic program and the structural equations of motion. The study included a single degree of freedom case with airfoil pitching at quarter chord and a three degree of freedom case with airfoil pitching, plunging and aileron pitching.

Mach numbers considered in [6] were 0.72 and 0.82, respectively. For the three degree of freedom case, results were obtained as plots of nondimensional pitching displacement (and moment), plunging displacement (and lift) and aileron pitching displacement (and moment) vs time for various arbitrary values of the airfoil-air mass ratios. The other aeroelastic parameters were also selected arbitrarily. All the displacement and force quantities in the figures were divided by  $\alpha'(0)$ , the initial pitching velocity. Values used for  $\alpha'(0)$  were 1, 3 and 5 degrees, respectively. By varying the values of the airfoil-air mass ratio, converging and diverging curves were obtained. It was, however, pointed out that no attempt was made to obtain response curves corresponding to flutter condition for the three degree of freedom system.

Transonic aeroelastic studies conducted so far ([3, 6, 9, 10]) for the two degree of freedom system (plunging and pitching) are only limited to flutter analysis. The aeroelastic response studies conducted have not included the single degree of freedom case of airfoil plunging and the two degree of freedom case. Although the single degree of freedom system does not represent the "real world" problem, studies of such case may provide insight to other practical aeroelastic problems. On the other hand, the two degree of freedom system can predict the flexure-torsion flutter which is the most destructive of all aeroelastic instabilities. Furthermore, the "transonic dip" phenomenon reported in [7-9] indicated that flexure torsion flutter speed may drop substantially in the transonic regime.

In this study, aeroelastic response analysis was performed for a flat plate and a NACA 64A006 airfoil. The cases considered include single pitching degree of freedom, single plunging degree of freedom, and two degrees of freedom. Aerodynamic forces were obtained by the use of the computer program LTRAN2[11] based on the time-integration method. Response results were obtained by numerically integrating the aeroelastic equations by a direct time integration method.

The present method was first evaluated by repeating an example in [5] and good agreement was found. The method was then evaluated by considering a linear flat plate oscillating with a single pitching and a single plunging degree of freedom at  $M_\infty = 0.7$ . Response results were obtained by using LTRAN2 and also by using the quasi steady-state aerodynamic theory. Both solutions were found in good agreement.

For the two degree of freedom case, a flat plate (at  $M_\infty = 0.7$ ) and a NACA 64A006 airfoil (at  $M_\infty = 0.85$ ) were considered. Time response curves for plunging displacement, lifting force, pitching rotation and pitching moment were obtained. By varying the airfoil-air mass ratio, these curves became neutrally stable, stable and unstable. The values of the parameters that produce neutrally stable responses were found to be in agreement with those obtained in separate flutter analyses.

#### AEROELASTIC EQUATIONS OF MOTION

In this study, only plunging and pitching motions of an airfoil are considered. The definitions of variables and sign conventions are described in Fig. 1 and Nomenclature.

In developing the following equations, it is assumed that the airfoil is rigid and the amplitude of oscillation is small. Since both the structural and aerodynamic equations are integrated simultaneously, it is not necessary to use the principle of linear superposition of airloads. The mean angle of attack is assumed as zero in all cases. It is also assumed that there is no coupling in mechanical damping.

##### (i) *Single pitching degree of freedom system*

The equation of motion for an airfoil pitching about an axis at a distance  $a_h b$  from the mid-chord is

$$I_a \ddot{\alpha} + C_a \dot{\alpha} + K_a \alpha = Q_a. \quad (1)$$

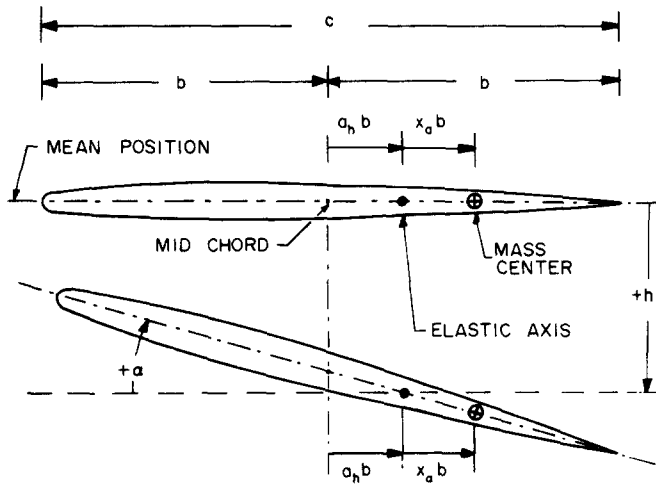


Fig. 1. Definition of parameters for two degree of freedom aeroelastic analysis.

A nondimensional form of (1) can be written as

$$\alpha'' + A_1 \alpha' + A_2 \alpha = \frac{Q_\alpha}{I_\alpha \omega^2} \quad (2)$$

where the prime represents the differentiation with respect to nondimensional time  $\omega t$ .

The aerodynamic moment  $Q_\alpha$  may be written as

$$Q_\alpha = q c^2 c_m \quad (3)$$

where the values for the aerodynamic moment coefficient  $c_m$  are obtained by using LTRAN2.

Using (3) and the definition  $k_c = \omega c / U$ , (2) can be written as

$$\alpha'' + A_1 \alpha' + A_2 \alpha = \frac{8}{\pi \mu' k_c^2} c_m \quad (4)$$

This equation is incorporated in LTRAN2 to obtain the aeroelastic responses.

If quasi steady-state aerodynamic theory is used, the aerodynamic moment  $Q_\alpha$  can be written as (from Chap. 5 of [2]),

$$Q_\alpha = \pi \rho b^2 \{ -U b (1/2 - a_h) \dot{\alpha} - b^2 (1/8 + a_h^2) \ddot{\alpha} \} \\ + 2 \pi \rho U b^2 (a_h + 1/2) \{ U \alpha + b (0.5 - a_h) \dot{\alpha} \}. \quad (5)$$

Correcting  $Q_\alpha$  for the effect of Mach number by multiplying the r.h.s. of (5) by Prandtl-Glauert number  $\beta$ , and substituting it into (2) yields.

$$d_1 \alpha'' + e_1 \alpha' + f_1 \alpha = 0 \quad (6)$$

where  $d_1 = 1.0 + [(0.125 + a_h^2) \beta / \mu']$ .

$$e_1 = A_1 + \frac{2(0.5 - a_h) \beta}{\mu' k_c} + \frac{4(a_h^2 - 0.25) \beta}{\mu' k_c}$$

$$f_1 = A_2 - \frac{8(a_h + 0.5) \beta}{\mu' k_c^2}.$$

In deriving (6), the correction for the effect of Mach number was made by using Prandtl-Glauert number  $\beta$  which is valid for steady state results. In the present study the effects of the terms containing  $\dot{\alpha}$  and  $\ddot{\alpha}$  of (5) on the response results were negligible when compared to that

of the term containing  $\alpha$  (see also page 279 of [2]). From eqn (5.346) of (2) it can be observed that the term containing  $\alpha$  can be independently obtained from steady state formulas. Hence Prandtl–Glauert number was used to correct  $Q_a$  for the effect of Mach number. This correction was required in order to obtain comparable results with LTRAN2.

Equation (6) is a second order, ordinary differential equation. From this equation, the closed form solution for pitching response can be obtained for given initial conditions and other variables.

(ii) *Single plunging degree of freedom system*

If  $h$  is the plunging displacement measured positive downwards from a mean position, the equation of motion for the system can be written as

$$m\ddot{h} + C_h\dot{h} + K_h h = Q_h. \quad (7)$$

The nondimensional form of (7) is written as

$$\delta'' + B_1\delta' + B_2\delta = \frac{Q_h}{m\omega^2 c} \quad (8)$$

where the prime represents the derivative with respect to nondimensional time  $\omega t$ .

The aerodynamic force  $Q_h$  may be expressed as

$$Q_h = -qcc_l \quad (9)$$

where  $c_l$  is the lift coefficient which can be obtained by LTRAN2. A negative sign is introduced in (9) because lift force is defined as positive upwards in the computer program.

Substituting (9) in (8) and rearranging gives

$$\delta'' + B_1\delta' + B_2\delta = -\frac{2c_l}{\pi\mu k_c^2}. \quad (10)$$

This equation is incorporated in LTRAN2 to obtain the aeroelastic responses for the plunging case.

If quasi steady-state aerodynamic theory is used, the aerodynamic lift  $Q_h$  can be written as (from Chap. 5 of [2])

$$Q_h = -2\pi U_p b \dot{h} - \pi \rho b^2 \ddot{h}. \quad (11)$$

Correcting  $Q_h$  for the effect of Mach number by multiplying the r.h.s. of (11) by Prandtl–Glauert number  $\beta$  and substituting it into (8) yields

$$d_2\delta'' + e_2\delta' + f_2\delta = 0 \quad (12)$$

where

$$d_2 = 1 + \frac{\beta}{\mu}, \quad e_2 = B_1 + \frac{4\beta}{\mu k_c}, \quad f_2 = B_2.$$

In deriving (12), the correction for effect of Mach number was made by using Prandtl–Glauert number  $\beta$  which is valid for steady state results. In the present study the effects of the term containing  $\ddot{h}$  of (11) is negligible on response results when compared to term containing  $\dot{h}$  (see page 279 of [2]). From eqn (5.345) of [2] it can be observed that the term containing  $\dot{h}$  can be independently obtained from steady state formulas. Hence Prandtl–Glauert number was used to correct  $Q_h$  for the effect of Mach number. This correction was required in order to obtain comparable results with LTRAN 2.

Equation (12) is a second order ordinary differential equation. From this equation, closed form solutions can be obtained for plunging responses for given initial conditions and other variables.

(iii) *Two degree of freedom system—pitching and plunging*

Considering the inertia forces, damping forces, elastic forces and aerodynamic forces, the equations of motion for a two degree of freedom system as shown in Fig. 1 are

$$m\ddot{h} + S\ddot{\alpha} + C_h\dot{h} + K_h h = Q_h \quad (13a)$$

$$S\ddot{h} + I_a\ddot{\alpha} + C_a\dot{\alpha} + K_a\alpha = Q_a. \quad (13b)$$

The nondimensional form of (13) can be written as

$$\xi'' + x_a\alpha'' + \zeta_h\xi' + \left[\frac{\omega_h}{\omega}\right]^2 \xi = \frac{Q_h}{mb\omega^2} \quad (14a)$$

$$x_a\xi'' + r_a^2\alpha'' + \zeta_a\alpha' + r_a^2\left[\frac{\omega_a}{\omega}\right]^2 \alpha = \frac{Q_a}{mb^2\omega^2} \quad (14b)$$

where the prime represents the derivative with respect to nondimensional time  $\omega t$ .

Substituting (3) and (9) into (14) and rearranging, the final matrix equation for the response analysis is

$$[M]\begin{Bmatrix} \xi'' \\ \alpha'' \end{Bmatrix} + [C]\begin{Bmatrix} \xi' \\ \alpha' \end{Bmatrix} + [K]\begin{Bmatrix} \xi \\ \alpha \end{Bmatrix} = \frac{4}{\pi\mu k_c^2} \begin{Bmatrix} -c_l \\ 2c_m \end{Bmatrix} \quad (15)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are the mass, damping and stiffness matrices, respectively, and they are defined as

$$[M] = \begin{bmatrix} 1 & x_a \\ x_a & r_a^2 \end{bmatrix}; [C] = \begin{bmatrix} \zeta_h & 0 \\ 0 & \zeta_a \end{bmatrix};$$

$$[K] = \left(\frac{2}{U^*K_c}\right)^2 \begin{bmatrix} (\omega_h/\omega_a)^2 & 0 \\ 0 & r_a^2 \end{bmatrix}. \quad (16)$$

The quantities  $c_l$  and  $c_m$  are directly obtained from LTRAN2.

Equation (15) is incorporated into LTRAN2 to obtain the aeroelastic responses for two dimensional airfoils with pitching and plunging degrees of freedom.

## RESPONSE SOLUTION PROCEDURE

The aeroelastic eqns (4), (10) and (15) can be written in general form as

$$[M]\{u''\} + [C]\{u'\} + [K]\{u\} = \{p\} \quad (17)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are the mass, damping, and stiffness matrices, respectively;  $\{u\}$  is the vector of the displacement degrees of freedom for the structure; prime denotes the derivatives with respect to nondimensional time  $t = \omega t$ ;  $\{p\}$  is the vector of the aerodynamic loads. Equation (17) is in the form suited for numerical integration.

Many numerical procedures are available in the literature for the solution of the equations of motion (17). Discussion of such procedures as applied to dynamic problems of structures can be found in references, such as [12]. In the present analysis the common direct integration method based on linear variation of acceleration is employed to find the time-history dynamic responses of the aeroelastic system.

The response solution is obtained by using a step-by-step time integration finite difference approach. This method is conditionally stable. It was found that the maximum time step required for LTRAN2 to obtain accurate aerodynamic results was actually smaller than that needed to give accurate results for structural response solution. However, in this study same time step was used for both aerodynamic and response computations. Thus, the time step was dictated by that needed in aerodynamic computation.

Within the values assumed for the time step in this study, it was observed that the direct

integration scheme is numerically stable. This was verified by repeating a response analysis for different time steps. The major advantage of the present scheme lies in its simplicity that only one aerodynamic computation is required for each time step.

In [6], aeroelastic response analysis was performed for one and three degree-of-freedom systems. It is stated that it was necessary to use a predictor-corrector approach in order to maintain stability. For the cases studied here, however, linear acceleration method was found to be sufficiently accurate and efficient.

Assuming a linear variation of acceleration, the velocities and displacements at the end of a small time interval  $\Delta \bar{t}$  can be expressed as

$$\{u'\}_{\bar{t}} = \{u'\}_{\bar{t}-\Delta \bar{t}} + \frac{\Delta \bar{t}}{2} \{u''\}_{\bar{t}-\Delta \bar{t}} + \frac{\Delta \bar{t}}{2} \{u''\}_{\bar{t}} \quad (18a)$$

$$\{u\}_{\bar{t}} = \{u\}_{\bar{t}-\Delta \bar{t}} + \Delta \bar{t} \{u'\}_{\bar{t}-\Delta \bar{t}} + \frac{\Delta \bar{t}^2}{3} \{u''\}_{\bar{t}-\Delta \bar{t}} + \frac{\Delta \bar{t}^2}{6} \{u''\}_{\bar{t}}. \quad (18b)$$

Substituting (18a) and (18b) into (17) yields

$$\{u''\}_{\bar{t}} = [F][\{p\}_{\bar{t}} - [C]\{v\} - [K]\{w\}] \quad (19)$$

where

$$[F] = \left[ [M] + \frac{\Delta \bar{t}}{2} [C] + \frac{\Delta \bar{t}^2}{6} [K] \right]^{-1} \quad (20a)$$

$$\{v\} = \{u'\}_{\bar{t}-\Delta \bar{t}} + \frac{\Delta \bar{t}}{2} \{u''\}_{\bar{t}-\Delta \bar{t}} \quad (20b)$$

$$\{w\} = \{u\}_{\bar{t}-\Delta \bar{t}} + \Delta \bar{t} \{u'\}_{\bar{t}-\Delta \bar{t}} + \frac{\Delta \bar{t}^2}{3} \{u''\}_{\bar{t}-\Delta \bar{t}}. \quad (20c)$$

For small amplitude (structurally linear) problems, matrix  $[F]$  need only be formed once since it is independent of time. On the other hand, the aerodynamic load vector  $\{p\}$  depends upon  $\{u\}$ ,  $\{u'\}$ ,  $k_c$  and  $M_\infty$ . The vector  $\{p\}$  is obtained by numerically solving the transonic aerodynamic equation with the use of LTRAN2. The values of  $\{u\}$  and  $\{u'\}$  used for computing  $\{p\}$  are based on the values obtained at the time  $\bar{t} - \Delta \bar{t}$ .

The step-by-step integration procedure for obtaining the aeroelastic response was carried out in the following manner. For a set of starting values of  $\{u\}$ ,  $\{u'\}$  and  $\{p\}$ , (say, known at time  $\bar{t} - \Delta \bar{t}$ ) the acceleration vector  $\{u''\}$  at time  $\bar{t}$  was computed from (19). Based on the known acceleration vector  $\{u''\}$ , the displacement vector  $\{u\}$  and velocity vector  $\{u'\}$  at time  $\bar{t}$  were computed from (18a) and (18b), respectively. From these quantities the effective induced angle of attack  $\alpha_i$ , and its time derivatives were computed for time  $\bar{t}$ . This value of  $\alpha_i$  and other required quantities were then read into the LTRAN2 code and the new aerodynamic load vector  $\{p\}$  at time  $\bar{t}$  was computed. At this stage all the quantities, namely,  $\{u\}$ ,  $\{u'\}$ ,  $\{u''\}$  and  $\{p\}$  at time  $\bar{t}$  were known so that further computations for time  $\bar{t} + \Delta \bar{t}$  can be carried out. This process was repeated for every time step. A flow chart of this process is shown in Fig. 2.

## RESULTS

Aeroelastic response analyses were carried out for two airfoil configurations, a flat plate and a NACA 64A006 airfoil. In both cases, the airfoils were considered as single and two degree of freedom systems.

In the aerodynamic computations, a 79 (vertical) by 99 (horizontal) finite difference mesh was employed. The mesh pattern near airfoil is shown in Fig. 3. The steady-state solution was obtained by using the successive line over-relaxation method (SLOR) and the unsteady computation was made by using time-integration method, both available in LTRAN2 code. Computations were carried-out on a CDC 6500 computer.

### (i) An NACA 64A006 airfoil pitching at $M_\infty = 0.88$

A case of an NACA 64A006 airfoil pitching about the mid-chord axis at  $M_\infty = 0.88$  and

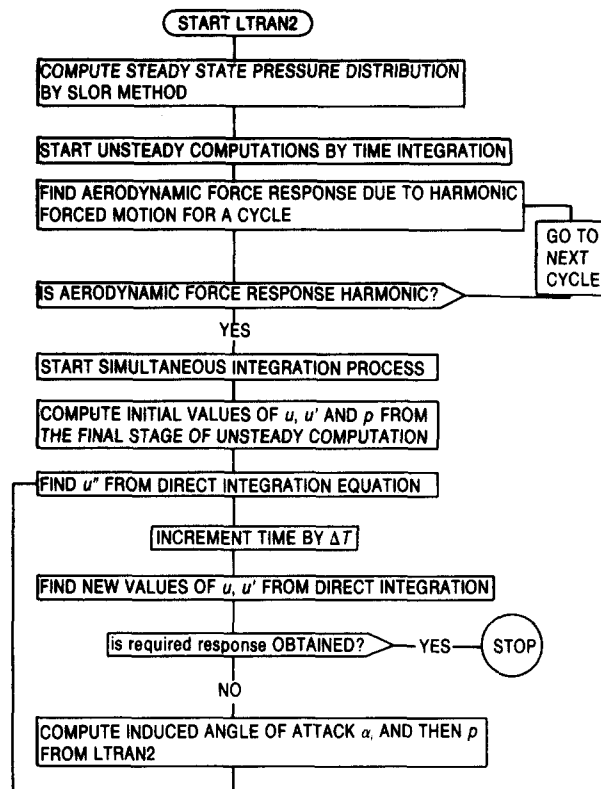


Fig. 2. Flow chart for time response analysis.

$k_c = 0.1$  was considered. This case was selected in order to verify the present method by comparing the results with those obtained by Ballhaus and Goorjian[5].

A detail report of such comparison was given in [13]. Good agreement was found. For example, the values of  $A_1$  and  $A_2$  in (4) that cause neutrally stable response were found in this study as 1.05 and 1.437, respectively. In [5],  $A_1$  and  $A_2$  were obtained as 1.072 and 1.414, respectively.

(ii) *A flat plate pitching at  $M_\infty = 0.7$*

A response analysis was performed for a flat plate pitching about mid-chord axis with  $k_c = 0.1$  and  $M_\infty = 0.7$ . The LTRAN2 (linear) results were compared with those obtained by quasi steady-state aerodynamic theory.

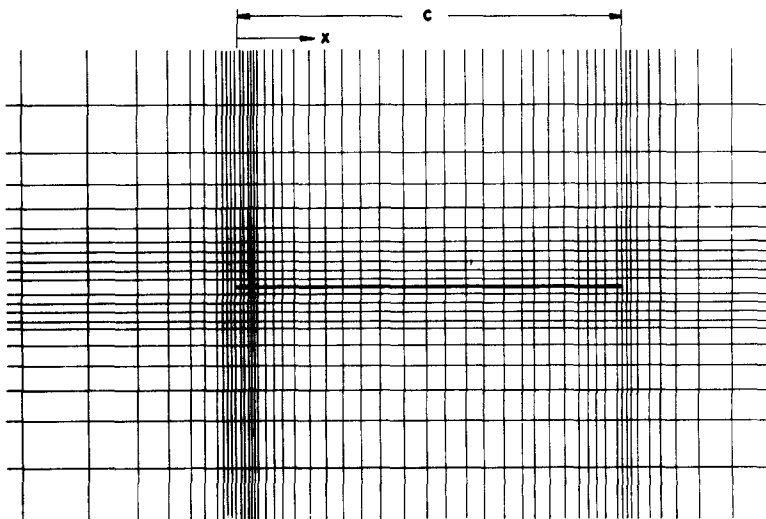


Fig. 3. Computational mesh near airfoil.



In this case, (4) is the governing differential equation of motion. The aerodynamic equation was integrated in time for four cycles by forcing a sinusoidal variation of pitching angle with amplitude of 0.01 rad. The free motion was started at the end of the fourth cycle. The initial conditions obtained for free motion were corresponding to  $\alpha(0) = 0.0$  and  $\alpha'(0) = 0.01$ . The structural parameters for free motion were so selected that a converging type response could be obtained. The values for the damping parameter  $A_1$ , the stiffness parameter  $A_2$ , and the airfoil-air moment of inertia ratio  $\mu'$  were 0.5, 1.5 and 1000, respectively. The value assumed for  $\mu'$  was quite high when compared to the actual values for aircraft wings, however, this number was required in order to obtain a response solution that can be compared with quasi steady-state theory.

The converging type response curves obtained for pitching angle  $\alpha$  and pitching moment  $c_m$  are shown in Fig. 4. In the same figure the responses obtained by employing the quasi steady-state aerodynamic theory are also shown. These responses were obtained by solving the differential eqn (6) for the same values of  $A_1$ ,  $A_2$ ,  $\mu'$ ,  $\alpha(0)$  and  $\alpha'(0)$  as used for LTRAN2. The two sets of solutions are, in general, in fairly good agreement.

Small differences in amplitude and phase angle in Fig. 4 between the two sets of results are mainly due to the difference between the two methods. It is observed that there is no phase lag between the pitching angle and the corresponding moment in the quasi steady-state solution but there is some phase lag in the LTRAN2 solution. Therefore, the effective aerodynamic dampings in the system between the two methods are different which may cause the discrepancy between the two sets of curves. Such discrepancy becomes larger for smaller values of  $\mu'$  since aerodynamic damping is inversely proportional to  $\mu'$ .

At this point, it may be noted that LTRAN2 is based on the low frequency approximation which is more valid for Mach numbers near unity. The discrepancy found in the two sets of curves may also be partly due to the low value of Mach number (0.7) considered.

### (iii) A flat plate plunging at $M_\infty = 0.7$

Response results were obtained for a flat plate with only a single plunging degree of freedom. The Mach number and reduced frequency  $k_c$  considered were equal to 0.7 and 0.1, respectively. Results were obtained both from LTRAN2 (linear) and quasi steady-state theory.

The governing differential equation of motion for this case is (10). The aerodynamic equation was first integrated by LTRAN2 for four cycles by forcing a sinusoidal plunging motion with amplitude of plunging displacement  $\delta = h/c = 0.1$ . This corresponds to an amplitude of 0.01 rad for the induced angle of attack  $\alpha_i = k_c \delta'$ .

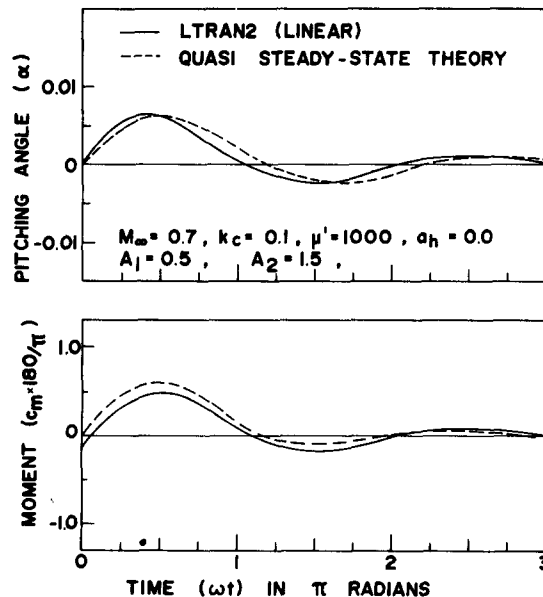


Fig. 4. Comparison of the response curves obtained by LTRAN2 and quasi steady-state theory for a flat plate pitching about mid-chord at  $M_\infty = 0.7$ .

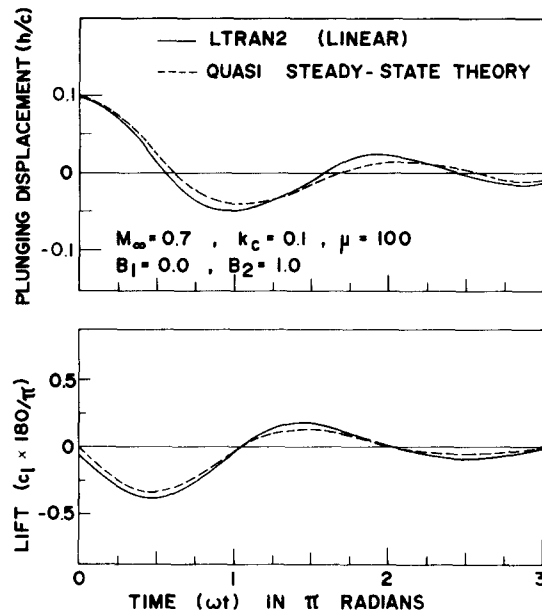


Fig. 5. Comparison of the response curves obtained by LTRAN2 and quasi steady-state theory for a flat plate plunging at  $M_\infty = 0.7$ .

The free motion was started with initial conditions corresponding to  $\alpha_i(0) = 0.0$  ( $\delta = 0.1$ ) and  $\alpha'_i(0) = 0.01$  ( $\delta' = 0.0$ ). The structural parameters were selected so that a converging type response could be obtained. The values assumed for the structural damping parameter  $B_1$ , the structural stiffness parameter  $B_2$  and the airfoil-air mass ratio  $\mu$  were equal to 0.0, 1.0 and 100, respectively. Response curves obtained from LTRAN2 for plunging displacement  $\delta$  and lifting force coefficient  $c_l$  are shown in Fig. 5. In the same figure the corresponding results obtained by quasi steady-state theory are also shown. These results were obtained by solving the differential eqn (12) based on the same initial conditions and structural parameters as used in LTRAN2.

Response results obtained by both methods compare fairly well. The small differences in phase angles and amplitudes may be mainly due to the difference in the methods.

(iv) *A flat plate plunging and pitching about mid-chord axis at  $M_\infty = 0.7$*

Responses were obtained for a flat plate plunging and pitching about mid-chord axis at Mach number  $M_\infty = 0.7$ . Both the LTRAN2 (linear) code and the Kernel Function method were used so that the two sets of results can be compared. Since this is a linear case good comparison can be expected.

Table 1. Aerodynamic coefficients for flat plate pitching about mid-chord axis at  $M_\infty = 0.7$

Aerodynamic Coefficient	Method*	$k_c = 0.05$		$k_c = 0.10$		$k_c = 0.15$	
		Real	Imag.	Real	Imag.	Real	Imag.
$c_{l_\delta}$	1	0.0669	0.4225	0.1700	0.8000	0.3038	1.134
	2	0.0616	0.3974	0.1666	0.7190	0.2719	0.9876
$c_{l_\alpha}$	1	8.449	-1.338	8.001	-1.701	7.557	-2.025
	2	7.964	-1.133	7.240	-1.488	6.671	-1.570
$c_{m_\delta}$	1	0.0151	0.0952	0.0479	0.1787	0.0954	0.2486
	2	0.0174	0.0990	0.0501	0.1780	0.0865	0.2425
$c_{m_\alpha}$	1	1.904	-0.3016	1.787	-0.4788	1.657	-0.6361
	2	1.985	-0.3500	1.794	-0.5117	1.640	-0.5996

\* Method 1: Time Integration by LTRAN2 (linear).

Method 2: Kernel Function Method.

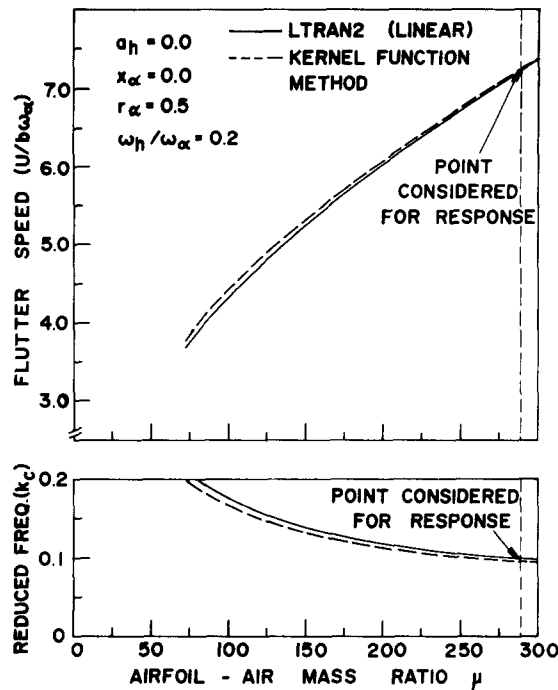


Fig. 6. Effect of airfoil-air mass ratio on flutter speed for a flat plate plunging and pitching about mid-chord axis at  $M_\infty = 0.7$ .

Before starting the response analysis it was necessary to find the aeroelastic parameters corresponding to a neutrally stable condition. Equation (15) is the governing equations of motion for this case. The aeroelastic parameters include:  $U^*$ ,  $\mu$ ,  $x_\alpha$ ,  $\omega_h/\omega_\alpha$ ,  $r_\alpha$ ,  $\zeta_h$ ,  $\zeta_\alpha$  and  $a_h$ . The values for  $x_\alpha$ ,  $\omega_h/\omega_\alpha$ ,  $r_\alpha$ ,  $\zeta_h$ ,  $\zeta_\alpha$  and  $a_h$  were assumed as 0.0, 0.2, 0.5, 0.0, 0.0 and 0.0, respectively.

In order to obtain the values of  $k_c$ ,  $U^*$  and  $\mu$  corresponding to the neutrally stable condition, a flutter analysis was performed [3]. The aerodynamic coefficients  $c_{l\beta}$ ,  $c_{l\alpha}$ ,  $c_{m\beta}$  and  $c_{m\alpha}$  were obtained by using the time integration method of LTRAN2 (linear) and the Kernel Function method for three values of reduced frequency  $k_c$  and are shown in Table 1. Agreement is good between the two methods. The  $U$ - $g$  method was used to perform the flutter analysis and the results for flutter speed and corresponding reduced frequency vs  $\mu$  are shown in Fig. 6. As expected the agreement is excellent between the two methods.

Response analyses were carried out for stable, neutrally stable and unstable conditions by selecting the aeroelastic parameters based on any selected point in the flutter speed curves. Because the time integration method was used in performing the response studies, it was essential that the flutter curves be obtained by using the time integration method instead of other methods such as the indicial and relaxation methods, etc.

The governing equation of motion for this case is (15). The aerodynamic equation was first integrated in time for four cycles by forcing a sinusoidal pitching motion with amplitude of 0.01 rad. In the fourth cycle the aerodynamic force responses became almost periodic. After that, the free motion was started by simultaneously integrating the structural and aerodynamic equations. This was started with initial conditions with  $\xi(0) = 0$ ,  $\xi'(0) = 0$ ,  $\alpha(0) = 0$  and  $\alpha'(0) = 0.01$ .

Aeroelastic parameters for the neutrally stable condition were selected from a point on the flutter speed curves in Fig. 6 at  $k_c = 0.1$ . The corresponding values of  $\mu$  and  $U^*$  were equal to 289.5 and 7.23, respectively. These values were substituted into (15) and the response analysis was carried out. Response curves for pitching angle  $\alpha$  and the corresponding pitching moment  $c_m$  for about six cycles are shown in Fig. 7. After small initial disturbances (due to initial conditions) both response curves show a perfect periodic behavior. Similar curves obtained for plunging displacement  $\xi$  and corresponding lifting force  $c_l$  are shown in Fig. 8. These curves also show a perfect periodic behavior after some small initial disturbances. It is seen that the neutrally stable conditions obtained by the present response method are in good agreement with those obtained in the flutter analyses based on both the time integration and the Kernel

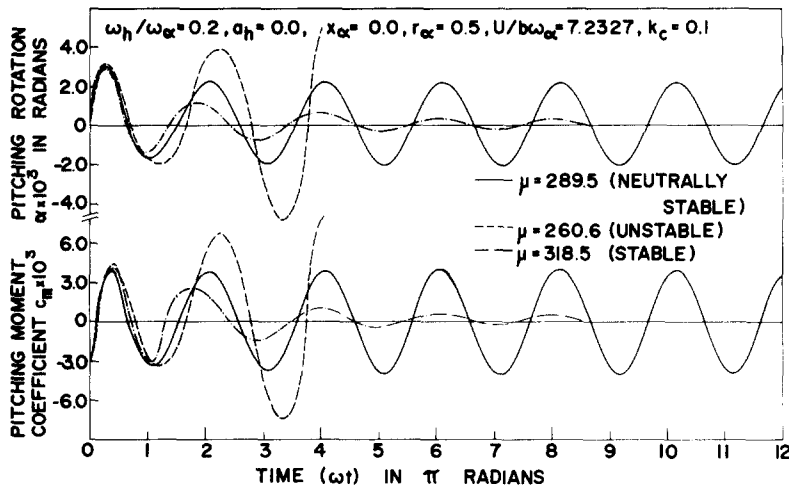


Fig. 7. Effect of airfoil-air mass ratio on response curves of rotation and moment for a flat plate plunging and pitching about mid-chord at  $M_\infty = 0.7$ .

Function method. In fact, as anticipated earlier, the results are exactly the same for the conditions chosen off the flutter curves using LTRAN 2 (linear).

In Fig. 7, the response curves for pitching displacement  $\alpha$  and pitching moment  $c_m$  for the stable and unstable conditions are also shown. These were obtained by changing the airfoil-air mass ratio  $\mu$ . For unstable response, the  $\mu$  value assumed was 10% lower than that corresponding to the neutrally stable condition. This point is in the unstable zone of Fig. 6. On the other hand, for stable response the  $\mu$  value selected was 10% higher than that corresponding to the neutrally stable condition. The corresponding stable and unstable responses for plunging displacement  $\xi$  and lifting force  $c_l$  are also shown in Fig. 8. It is seen that the stable and unstable conditions obtained in the flutter analysis produced converging and diverging responses, respectively.

It was also of interest to study the effect of airfoil-air mass ratio (altitude)  $\mu$  on the peak amplitudes of the response curves. In this analysis, the peak amplitudes corresponding to the second free cycle were considered. Figure 9 shows plots of the ratio of the second cycle peak amplitude to the amplitude of the neutrally stable curve vs the ratio of  $\mu$  to its value at the neutrally stable condition. The plots include four curves for plunging displacement, lifting force, pitching displacement and pitching moment, respectively. The last two curves coincide.

(v) *An NACA 64A006 airfoil plunging and pitching about 1/4-chord axis at  $M_\infty = 0.85$*

In the flutter analysis performed in [9], a "transonic dip" phenomenon was observed for the

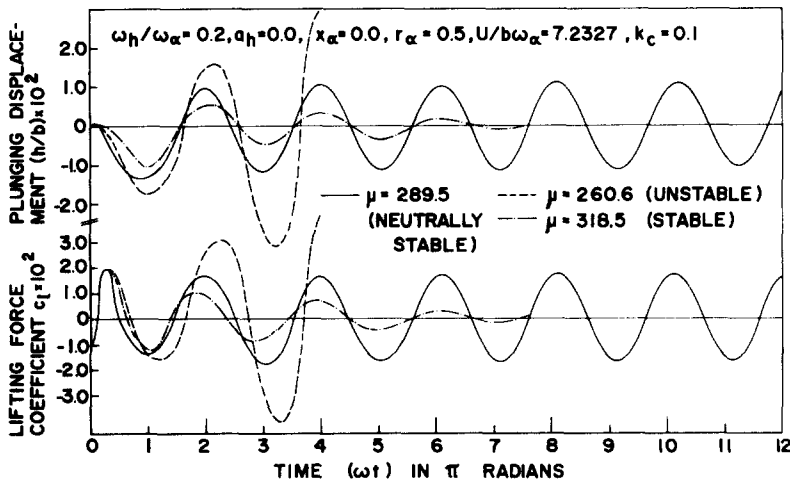


Fig. 8. Effect of airfoil-air mass ratio on response curves of displacement and lift for a flat plate plunging and pitching about mid-chord at  $M_\infty = 0.7$ .

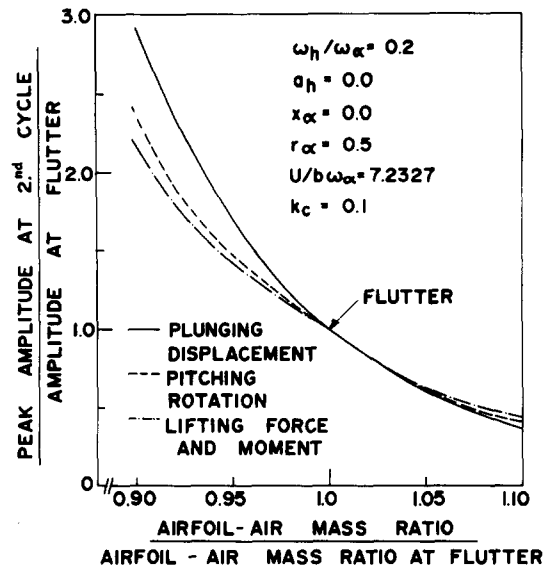


Fig. 9. Effect of airfoil-air mass ratio on peak amplitudes of the response curves for the flat plate case.

NACA 64A006 airfoil. Flutter speeds reached a minimum in the neighborhood of  $M_\infty = 0.85$ . In this study aeroelastic response analysis was carried out for this critical value (0.85) of Mach number.

Using the steady-state initial conditions, unsteady computations were carried out by using LTRAN2 (nonlinear). The time integration method was employed to obtain the aerodynamic coefficients  $c_{l\delta}$ ,  $c_{l\alpha}$ ,  $c_{m\delta}$  and  $c_{m\alpha}$ . The coefficients for three values of reduced frequency  $k_c$  are shown in Table 2. In the same table, the aerodynamic coefficients computed by UTRANS2 and the indicial method in [3] are also given. In general, all three methods agree well. It may be noted that the results obtained by the time integration method are expected to be more accurate than those obtained by the other two methods for  $k_c < 0.2$ .

Based on Table 2, flutter analysis was performed by using the method given in [3]. Plots of

Table 2. Aerodynamic coefficients for NACA 64A006 airfoil pitching about 1/4-chord axis at  $M_\infty = 0.85$

Aerodynamic Coefficient	Method*	$k_c = 0.05$		$k_c = 0.10$		$k_c = 0.15$	
		Real	Imag.	Real	Imag.	Real	Imag.
$c_{l\delta}$	1	0.302	0.678	0.616	1.067	0.867	1.335
	2	0.136	0.666	0.492	1.180	0.938	1.470
	3	0.163	0.626	0.480	0.962	0.783	1.148
$c_{l\alpha}$	1	13.561	-6.038	10.669	-6.160	8.900	-5.780
	2	13.310	-2.720	11.820	-4.920	9.770	-6.250
	3	11.803	-3.701	9.160	-4.491	7.223	-4.076
$c_{m\delta}$	1	0.010	-0.038	0.011	-0.068	-0.028	-0.104
	2	-0.001	-0.034	-0.005	-0.068	-0.012	-0.102
	3	0.004	-0.026	0.009	-0.054	0.013	-0.089
$c_{m\alpha}$	1	-0.749	-0.201	-0.675	-0.107	-0.691	0.185
	2	-0.676	0.021	-0.677	0.053	-0.677	0.082
	3	-0.501	-0.071	-0.510	-0.147	-0.514	-0.210

\* Method 1: LTRAN2 Time Integration (79 x 99)  
 Method 2: LTRAN2 Indicial Method (79 x 99)  
 Method 3: STRANS2 and UTRANS2 Relaxation method (59 x 60)

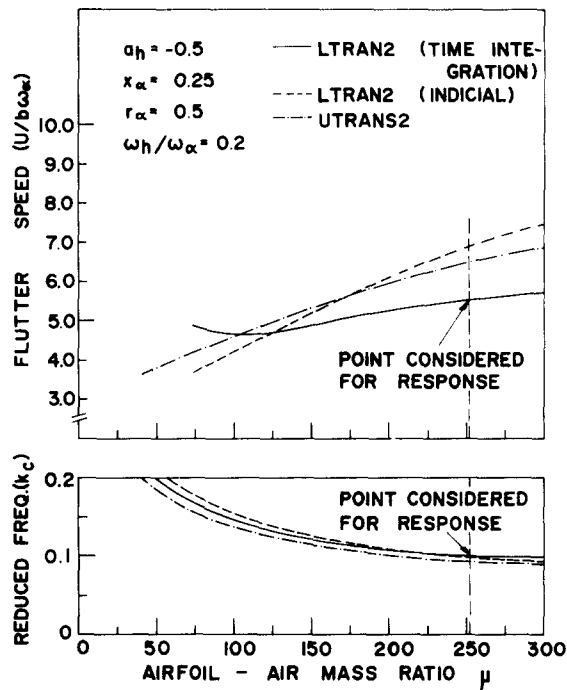


Fig. 10. Effect of airfoil-air mass ratio on flutter speed for NACA 64A006 airfoil plunging and pitching at  $M_\infty = 0.85$ .

flutter speed and the corresponding reduced frequency vs airfoil-air mass ratio are shown in Fig. 10. The values assumed for  $a_h$ ,  $x_\alpha$ ,  $\omega_h/\omega_\alpha$ ,  $r_\alpha$ ,  $\zeta_h$  and  $\zeta_\alpha$  were  $-0.5$ ,  $0.25$ ,  $0.2$ ,  $0.5$ ,  $0$  and  $0$ , respectively.

It is seen in Fig. 10 that the agreement in flutter speed among the three methods is not so good. This may be due to the assumption of time linearization in the indicial and relaxation method. Furthermore, the provision of treating the shock wave motion is lacking in the relaxation and the indicial methods.

In the response analysis, (15) is the governing aeroelastic equation of the system. Reduced frequency was assumed as  $k_c = 0.1$ . The aerodynamic equation was first integrated in time for six cycles by forcing a sinusoidal pitching motion with amplitude of  $0.01$  rad. In the sixth cycle the forced responses in LTRAN2 became almost periodic. From this point onwards free motion was considered by simultaneously integrating the aerodynamic and structural equations. Free motion was started with initial conditions with  $\xi(0) = 0$ ,  $\xi'(0) = 0$ ,  $\alpha(0) = 0$  and  $\alpha'(0) = 0.01$ .

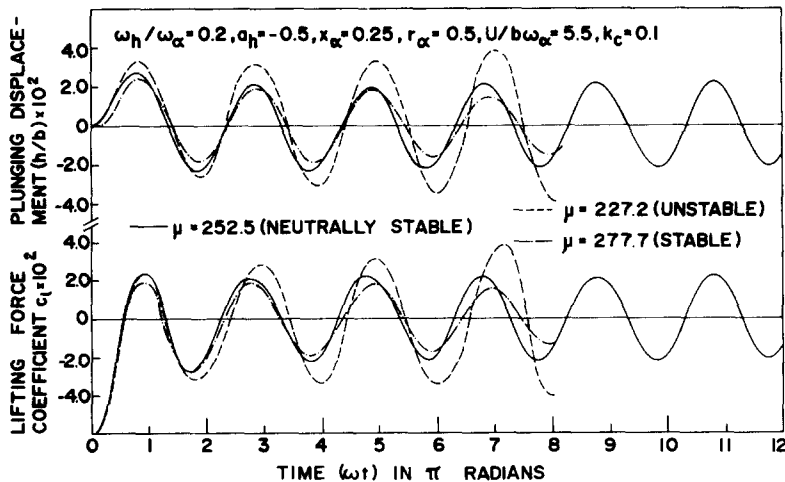


Fig. 11. Effect of airfoil-air mass ratio on response curves of displacement and lift for a NACA 64A006 airfoil plunging and pitching at  $M_\infty = 0.85$ .

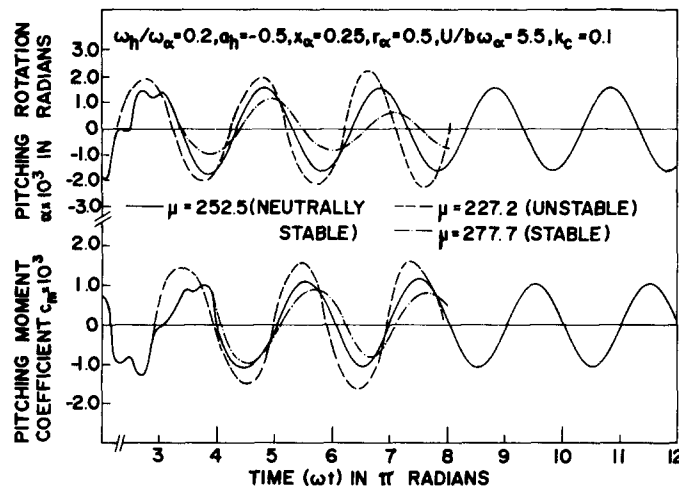


Fig. 12. Effect of airfoil-air mass ratio on response curves of rotation and moment for a NACA 64A006 airfoil plunging and pitching at  $M_\infty = 0.85$ .

A neutrally stable response was obtained by considering a point on the flutter speed curve obtained by the time integration method in Fig. 10 at  $k_c = 0.1$ . The corresponding values of  $\mu$  and  $U^*$  thus obtained were equal to 252.5 and 5.5, respectively. These values along with the other aeroelastic parameters ( $a_h = -0.5$ ,  $x_\alpha = 0.25$ ,  $\omega_h/\omega_\alpha = 0.2$ ,  $r_\alpha = 0.5$ ,  $k_c = 0.1$ ) were substituted into (15) and the response analysis was carried out. The responses for plunging displacement and corresponding lifting force obtained from LTRAN2 are shown in Fig. 11. After some small initial disturbances due to the initial conditions, the response curves shown ideal periodic behavior.

The corresponding stable response curves for pitching displacement  $\alpha$  and pitching moment  $c_m$  are shown in Fig. 12. These curves are shown only from the second cycle of the free motion. As compared to Fig. 11, there were quite pronounced initial disturbances due to initial conditions in the first cycle. However, the responses became almost periodic after the third cycle of free motion.

The curves for neutrally stable responses presented in Figs. 11 and 12 show that the neutrally stable conditions obtained by the present response method are in good agreement with those obtained in the flutter analysis based on the time integration method. In general, such ideal periodic behavior can not be expected since the aerodynamics are nonlinear now. On the other hand, the flutter points predicted by the indicial and relaxation methods will result in a slightly diverging case.

The response results for the neutrally stable condition presented in Figs. 11 and 12 also show the validity of the principle of superposition of air loads which was used in the flutter analysis. In deriving the equations for the simultaneous integration procedure it was not necessary to use such assumption. A possible reason for this fact may be for the case considered the lift and moment responses were harmonic. However, further testing needs to be done to check the validity of the superposition, especially for cases in which the lift and moment responses contain higher harmonics (nonlinear).

In Fig. 11, the response curves for plunging displacement  $\xi$  and lifting force  $c_l$  for stable and unstable conditions are also shown. These were obtained by changing the airfoil-air mass ratio  $\mu$ . For unstable response, the  $\mu$  value assumed was 10% less than that corresponding to the neutrally stable condition. This point is in the unstable zone of Fig. 10. On the other hand, for stable response the value of  $\mu$  assumed was 10% higher than that corresponding to the neutrally stable condition. This point is in the stable zone of Fig. 10. Stable and unstable response curves obtained for pitching displacement  $\alpha$  and pitching moment  $c_m$  are also shown in Fig. 12. It is seen that the stable and unstable conditions obtained in the flutter analysis produced converging and diverging responses, respectively.

It was also of interest to study the effect of airfoil-air mass ratio  $\mu$  (altitude) on peak amplitudes of response curves. In this analysis, the peak amplitudes corresponding to the fourth cycle of free motion were considered. Fig. 13 shows plots of the ratio of fourth cycle peak

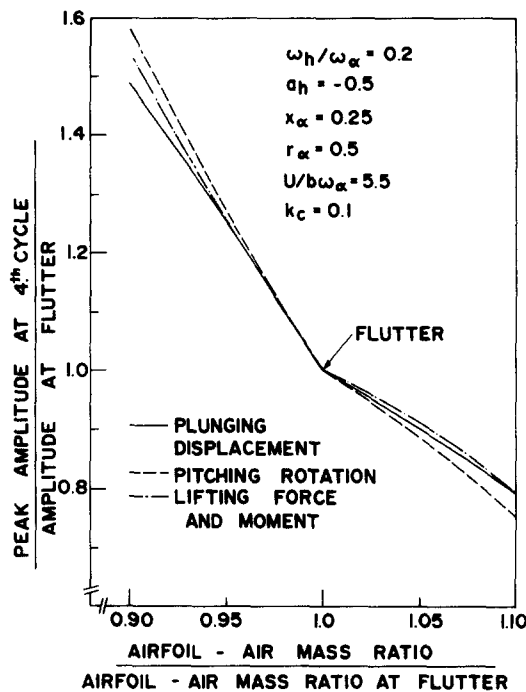


Fig. 13. Effect of airfoil-air mass ratio on peak amplitudes of the response curves for the NACA 64A006 case.

amplitude to the amplitude of the neutrally stable curves vs the ratio of  $\mu$  to its value at the neutrally stable point. These plots include four curves for plunging displacement, lifting force, pitching displacement and pitching moment, respectively. The last two curves coincide.

(vi) *Procedure for finding accurately the transonic flutter boundaries*

In this study a procedure to integrate structural and transonic aerodynamics equations was presented. This procedure can be used to find accurately the transonic flutter boundaries of single and two degree of freedom systems as follows.

(1) For a given aeroelastic system, the unsteady coefficients are found either by indicial or time integration method. Indicial method is computationally more economical than time integration method.

(2) Based on the unsteady aerodynamic coefficients flutter boundary is found by  $U-g$  method[3]. The flutter boundary obtained by using time integration aerodynamic coefficients may be closer to exact flutter boundary than that obtained by the indicial method.

(3) Based on aeroelastic parameters corresponding to the flutter boundary given by  $U-g$  method time response analysis is performed to find displacements and forces.

(4) If the aeroelastic parameters predicted by  $U-g$  method are exact, perfect periodic responses will be obtained at step (3). In general, this will not be the case in transonic regime. It is necessary to check whether the responses are stable or unstable.

(5) With the help of flutter boundary curve obtained by  $U-g$  method the aeroelastic parameters are suitably perturbed such that they are closer to neutrally stable condition.

(6) Based on new set of aeroelastic parameters the time response analysis is again performed.

(7) Steps (5) and (6) are repeated till satisfactory periodic time response results are obtained.

#### CONCLUDING REMARKS

(1) The response results obtained for single degree of freedom systems of pitching and plunging for flat plates at  $M_\infty = 0.7$  show that LTRAN2 (linear) results compare well with the results based on quasi steady-state aerodynamic theory.



(2) Results obtained for a flat plate with two degrees of freedom at  $M_\infty = 0.7$  illustrate that neutrally stable conditions obtained in the flutter analysis based on aerodynamic coefficients computed by the time integration method (LTRAN2) check with the neutrally stable conditions obtained by the time-response method. It was also illustrated that the neutrally stable conditions obtained in the flutter analysis based on aerodynamic coefficients obtained by the Kernel Function method compare well with those obtained by the time-response method.

(3) Response results obtained for a NACA 64A006 airfoil with two degrees of freedom at  $M_\infty = 0.85$  show that neutrally stable conditions obtained in the flutter analysis based on aerodynamic coefficients computed by the time integration method (LTRAN2) agree well with those obtained by the time integration response analysis. However, the neutrally stable conditions obtained in the flutter based on the relaxation and indicial aerodynamic coefficients do not provide such good agreement. The lack of good agreement may be due to the assumption of time linearization in the indicial and relaxation methods. Furthermore, the provision of treating the shock wave motion is lacking in the relaxation and the indicial methods.

(4) Good agreement between the neutrally stable conditions obtained in the flutter analysis (based on time integration aerodynamic coefficients) and the time-response analysis indicate that the principle of linear superposition of airloads used in the flutter analysis is valid for the cases analyzed. A possible reason for this fact may be that for the cases considered the lift and moment responses are simple harmonic. However, further testing needs to be done to check the validity of the principle of superposition, especially for cases in which the lift and moment responses contain higher harmonics (nonlinear).

(5) In general it may be concluded that the present method is accurate for predicting the neutrally stable conditions (flutter) for a two degree of freedom system. This method also takes into consideration the movement of the shock whereas it is not possible to do so in a pure flutter analysis. Thus, the present method can provide a comparison to experimental flutter studies.

(6) The present time response algorithm requires about 500 sec of CDC 6500 CPU time for one cycle.

(7) For the  $(79 \times 99)$  mesh used in this study, 150,000 words of core memory was required on CDC 6500 computer.

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## REFERENCES

1. W. F. Ballhaus and J. O. Bridgeman, *Numerical Solution Techniques for Unsteady Transonic Problems*. AGARD Report No. 679, Paper No. 16 (1980).
2. R. L. Bisplinghoff, H. Ashley and R. L. Halfman, *Aeroelasticity*. Addison Wesley, Reading, Massachusetts (1955).
3. T. Y. Yang, A. G. Striz and P. Guruswamy, *Flutter Analysis of Two-Dimensional and Two Degree of Freedom Airfoils in Small-Disturbance Unsteady Transonic Flow*. Air Force Flight Dynamics Laboratory, TR-78-202 (1978).
4. H. Ashley, On the role of shocks in the "Sub-transonic" flutter phenomenon. *Proc. AIAA/ASME/ASCE/AHS 20th Structures, Structural Dynamics and Materials Conf.* St. Louis, Missouri (1979).
5. W. F. Ballhaus and P. M. Goorjian, Computation of unsteady transonic flows by the indicial method. *AIAA J.* 16(2), 117-124 (1978).
6. D. P. Rizzetta, Time-dependent responses of a two-dimensional airfoil in transonic flow. *AIAA J.* 17(1), 26-32 (1979).
7. R. M. Traci, E. D. Albano and J. L. Farr, *Small Disturbance Transonic Flows About Oscillating Airfoils and Planar Wings*. Air Force Flight Dynamics Laboratory, TR-75-100 (1975).
8. M. G. Farmer and P. W. Hanson, Comparison of supercritical and conventional wing flutter characteristics. *Proc. AIAA/ASME/SAE 17th Structures, Structural Dynamics and Materials Conf.* King of Prussia, Pennsylvania, 608-611 (1976).
9. T. Y. Yang, P. Guruswamy, A. G. Striz and J. J. Olsen, Flutter analysis of a NACA 64A006 airfoil in small disturbance transonic flow. *J. Aircraft* 17(4), 225-232 (1980).
10. D. P. Rizzetta, *Transonic Flutter Analysis of a Two-Dimensional Airfoil*. Air Force Flight Dynamics Laboratory, TM-77-64-FBR (1977).
11. W. F. Ballhaus and P. M. Goorjian, Implicit finite difference computations of unsteady transonic flows about airfoils. *AIAA J.* 15(1), 1728-1735 (1977).
12. K. J. Bathe and E. L. Wilson, *Numerical Methods in Finite Element Analysis*, (Chaps. 8 and 9). Prentice-Hall, Englewood Cliffs, New Jersey (1976).
13. T. U. Yang, P. Guruswamy and A. G. Striz, *Aeroelastic Response Analysis of Two-Dimensional, Single and Two Degree of Freedom Airfoils in Low Frequency, Small Disturbance Unsteady Transonic Flow*. Air Force Flight Dynamics Laboratory, TR-79-3077 (1979).